Stat-Teaser

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Bad Leverages?

Pat's Stat Tips

Question: Why do some of my leverages get flagged as "bad" after reducing my model?

Answer: They may get flagged, but they're not bad!

Leverage is the amount of potential each design point exerts based on its position in the design space and the model being fit. The maximum leverage any point can have is 1. (The maximum leverage a replicated point can have is one divided by the number of replicates.) To illustrate, let's look at a linear model fit to two unique design points, one of which is replicated.

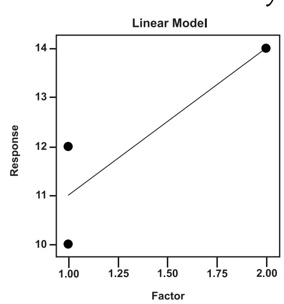


Figure 1: Plot of Linear Model

The two replicates of factor level 1 each have a leverage of 0.5 and the model fits their average. The single replicate of factor level 2 has a leverage of 1.0 and is fit perfectly. A leverage of one implies a point must be fit perfectly by the model and any error associated with the point is built into the model. For this reason, high leverage points are not desirable. In the case statistics section of the ANOVA report, Design-Ease® and Design-Expert® software "flag" points with leverages greater than two times the average leverage. But if leverages get flagged after model reduction, don't be overly concerned.

Let's illustrate the principles of leverage using a two-factor, face-centered Central Composite Design (CCD). This is a response surface design composed of 1) factorial points — located at the corners of the square to give information about linear and interaction coefficients; 2) axial points (star points) — these give information about the pure quadratic and linear coefficients; and 3) center points (replicated) — these give information about the pure quadratic coefficients.

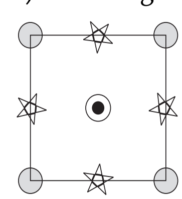


Figure 2: Face-Centered CCD

We'll analyze the response shown on table 3 and fit the full quadratic model. The statistical results are shown below:

Factor	Coefficient Estimate	t for H_o Coeff=0	Prob>|t|
Intercept	99.31		
A-Factor A	-5.44	-14.65	<0.0001
B-Factor B	7.79	20.98	<0.0001
A^2	0.51	0.93	0.3810
B^2	0.49	0.89	0.4034
AB	-11.37	-25.01	<0.0001

Table 1: Full Quadratic Model

— *continued on page 4*

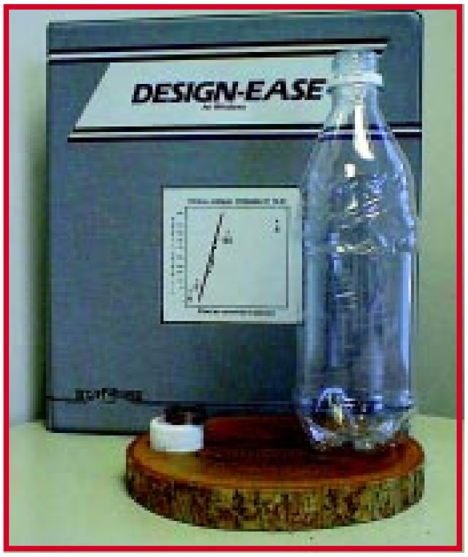
Mysterious March of the Sugar-Pop Soldiers

Mark's Experiment

by Mark J. Anderson

"It's eerie," my colleague John Guerin said when I demonstrated how long a plastic pop bottle will wobble when placed upon certain surfaces. While John taught his portion of a class on DOE, I was experimenting in the back with an empty root-beer bottle, killing time — at 25 seconds a wobble to be precise. (Engineers are easily amused!)

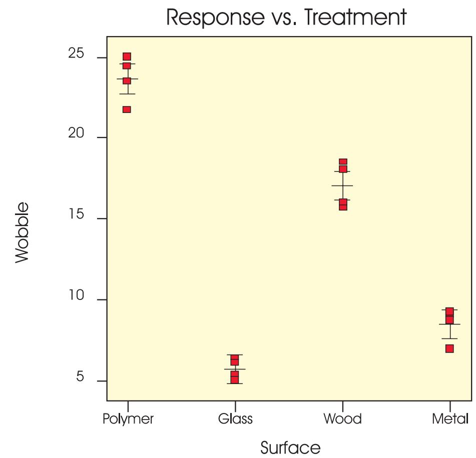
You've no doubt drunk from one of these bottles. They've got an unusual star-shaped bottom with five blunt



points. I can't explain why they're shaped so strangely, but I can tell you that they're very unstable when emptied. This is not a trivial issue in the Anderson home because we've got tons of teenagers hanging around guzzling caffeinated colas. The empty bottles multiply in Tribble-like fashion. The vibrations from my son's multi-megawatt sub-woofer bring these "dead soldiers" back to life. They wobble drunkenly across the countertops and over the edge, dribbling their syrupy residual on the floor. Yuk!

After class I tried to reproduce the astounding wobble of the plastic pop bottle. I thought I might do a two-level DOE on various amounts of residual liquid, tip height, cap weight, and the like. However, no matter how hard I tried, I could not get the bottle to wobble more than five seconds. Then I realized that it was the surface that made the difference. I was doing the experiment on a glass coffee table, whereas the table in class was clad in a polymeric material. Sure enough, when I tipped the bottle on the similarly surfaced bathroom counter, I got the complete 25-second dance. This led me to experiment on other surfaces: metal and wood.

The results from five trials each are shown on the effects graph from Design-Ease® software. Notice that the least significant difference (LSD) bars do not overlap. Therefore, it can be said with greater than 95 percent confidence that all sur-



faces differed. The order in decreasing effect is polymer > wood >> metal > glass.

Of course these surfaces were all unique in terms of smoothness, thickness, and other material properties. I have no idea why they should interact so differently with the bottle.

My data came from a 20-ounce PETE-plastic bottle tipped from an eight-millimeter height (the width of four quarters). Next time you're stuck in a boring meeting or delayed at the airport, check it out for yourself. In Minneapolis and several other terminals you can go to the Cheer's bar and order a bottle. For inspiration you can sit next to the life-size dummy of Cliff Clavin, the nerdy postman. Perhaps he'll explain the wobbling as follows: "This here is a phenomenon caused when the elastic moduli match, which creates a harmonic wave, dampened in various ways by ambient temperature and humidity, etc., etc., etc."

Any other theories?

— Mark Anderson (e-mail: mark@statease.com)

Real Life DOE Workshop!

Our newest class, designed by student request, teaches you how to deal with difficult data analysis decisions such as:

- What should you do when you have several outliers?
- How can proper residual analysis help you salvage data?
- How can my data be interpreted best?

All of these questions and more will be answered in our "Real Life DOE" workshop! The computer-intensive format leads you through the most interesting and informative cases from the extensive Stat-Ease files. Students are encouraged to bring their own data for specialized analysis.

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Introducing the Stat-Ease Crew

Front Row (L to R): Brian Smith, Tryg Helseth, Carol Summer, Neal Vaughn, Jim Whitcomb. Back Row (L to R): Pat Whitcomb, Sherry Brostrom, Shari Kraber, Gina Herrman, Roxanne Murray, Tim Gogolin, Carl Polnaszek, Mark Anderson.

Stat-Ease Does Dallas!

Stat-Ease, Inc. will be visiting the great state of Texas several times this year!

On March 29, Mark Anderson has been invited to speak at the American Chemical Society National Conference (see the "Where Can You Find Us?" column). On May 5–8 we are conducting a public workshop, "Experiment Design Made Easy," co-taught by Shari Kraber and Carl McAfee. If you want to sign up for this workshop, call Carol at **800-801-7191, ext. 18**. On August 9–13 we will present two talks at the Joint Statistical Meetings hosted by the American Statistical Association.

If you would like more information about any of these activities, please call us or send e-mail to **info@statease.com**.

Where Can You Find Us?

March '98

- American Institute of Chemical Engineers National Meeting, New Orleans: **"Statistical DOE for Improvement of Fertilizer Products"**
- American Chemical Society National Conference, Poster Session, Dallas *(Booth 946)*:**"Computer Aided DOE for Mixtures"**

April '98

- 18th Quality Expo, Chicago *(Booth 20059)*: **"Real Life DOE"**

May '98

- 51st Annual Quality Congress, Philadelphia *(Booth 605)*: **"Keys to Successful Designed Experiments"**

August '98

- Joint Statistical Meetings, Dallas *(Booth to be announced)*: **"Teaching Aids for DOE"** and **"DOE on Paraffin Blend for Therapeutic Bath"**

Invite us to your regional or national conference. Send e-mail to mark@statease.com

Reader Response to Mark's Shade Experiment

In the December 1997 issue of the Stat-Teaser, *Mark reported that a shiny exterior was best for deflecting the bright sun. Now, a reader tells us why.*

People who make and use telescopes try to keep their equipment from cooling down too much under clear cold night skies. (When it is too cold, dew forms.) The basic physics are similar to the hot or cold car problem. One factor that turns out to be important is the emissivity of the surface. A shiny metallic surface has low absorptivity (most of the heat reflects off) but also has low emissivity (does not radiate much heat away). A bright white surface also has low absorptivity, but has high emissivity (radiates a lot of heat away).

On a clear sunny day, the sun is a very bright heat source, but the sky away from the sun is at a very low radiation temperature. If you can arrange to reflect, or otherwise keep the sun's direct radiation away, you can achieve a fair amount of cooling by arranging for a high emissivity surface to radiate to the open sky.

At night, without the sun as a heat source, a black or white object will actually get colder than a shiny metallic object. The high emissivity of the black or white surface radiates heat away to the cold sky. A low emissivity, shiny surface radiates less heat to the cold sky, so stays warmer. At night, an umbrella, or other "shade" which protects from the open sky, actually keeps objects warmer.

— Mark D. Holm,
Aristech Chemical Corp.

— *continued from page 1*

The Prob>|t| values for the pure quadratic coefficients (A^2 and B^2) are not significant since they are greater than 0.10. However, let's leave these terms in the model and look at the 3D view of the

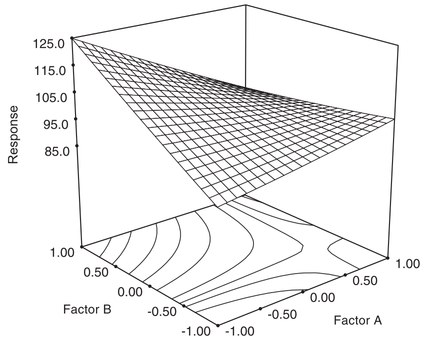

Figure 3: 3D Surface of Full Quadratic Fit

model graph (figure 3). Now delete the pure quadratic terms, A^2 and B^2, leaving a factorial model:

Factor	Coefficient Estimate	t for H_o Coeff=0	Prob>|t|
Intercept	99.77		
A-Factor A	-5.44	-14.12	<0.0001
B-Factor B	7.79	20.22	<0.0001
AB	-11.37	-24.11	<0.0001

Table 2: Reduced Model (Factorial)

The fit using the reduced (factorial) model is as good as with the full quadratic. Figure 4, the 3D surface for reduced quadratic, supports this statement. You can see that it is nearly identical to figure 3. However, you may be misled by changes in leverage. Because the ratio of coefficients to points decreases with model reduction, ALL the leverages are

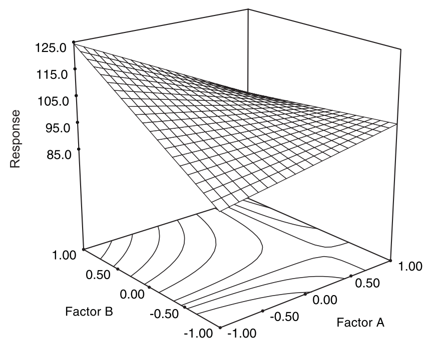

Figure 4: 3D Surface of Reduced Model (Factorial)

Table 3: Table of Leverages (# Flagged)

Std	Type	A	B	Response	Leverage Full Model (Quadratic)	Leverage Reduced (Factorial)
1	Factorial	-1.0	-1.0	85.9	0.79	0.66 #
2	Factorial	+1.0	-1.0	98.8	0.79	0.66 #
3	Factorial	-1.0	+1.0	124.4	0.79	0.66 #
4	Factorial	+1.0	+1.0	91.9	0.79	0.66 #
5	Axial	-1.0	0.0	106.5	0.49	0.24
6	Axial	+1.0	0.0	93.4	0.49	0.24
7	Axial	0.0	-1.0	92.3	0.49	0.24
8	Axial	0.0	+1.0	107.5	0.49	0.24
9	Center	0.0	0.0	98.4	0.17	0.077
10	Center	0.0	0.0	99.0	0.17	0.077
11	Center	0.0	0.0	100.2	0.17	0.077
12	Center	0.0	0.0	99.1	0.17	0.077
13	Center	0.0	0.0	99.6	0.17	0.077
				Average Leverage:	0.46	0.31

reduced, some more than others. The leverages of axial and center points are greatly reduced when the quadratic coefficients are eliminated (this is good!), but the leverages of the factorial points are hardly affected.

However, after model reduction, the factorial point leverages (std runs 1–4) are now greater than two times the average, so they get flagged. However, they are still less than for the previous full quadratic model. Therefore, in this case, they may be flagged, but they're not bad.

— *Pat Whitcomb (e-mail: pat@statease.com)*

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