Magic of multifactor testing revealed by fun physics experiment

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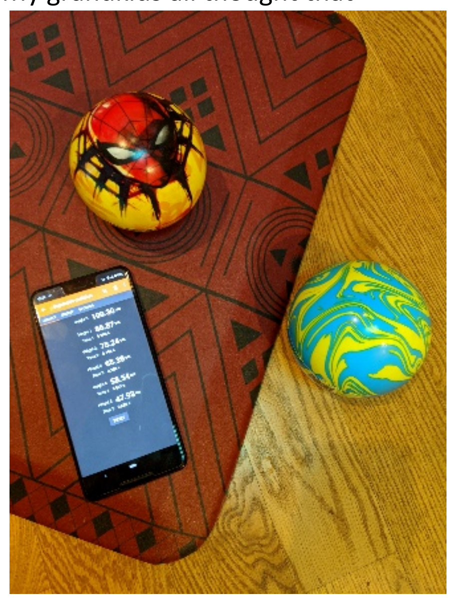
Part One—Setup

The behavior of elastic spheres caught my attention due to a proposed, but not completed, experiment on ball bounciness turned in by a student from the South Dakota School of Mines and Technology.* I decided to see for myself what would happen.

To start, I went shopping for suitable elastic spheres. As pictured, I found two ball-toys with the same diameter—one of them with an eye-catching Spider-Man graphic. My grandkids all thought that "Spidey" would bounce higher than the other ball—the one in swirly blue and yellow. Little did they know just by looking that "Swirley" was the one with superpowers, it being made from exceptionally elastic, solid synthetic rubber. Sadly, Spidey turned out to be a hollow airhead. This became immediately obvious when I dropped the two balls side by side from shoulder height. Spidey rebounded only to my knee while Swirley shot all the way back to nearly to the original drop level. This really amazed the children.



My next idea for this experiment came from Frugal Fun for Boys and Girls, a website that provides many great science projects. Their bouncy ball experiment focuses on the effect of temperature. However, I could see one big problem straight away: How can you get an accurate measure of bounce height? That led me an amazing cell-phone app called Phyphox (Physics Phone Experiments). Via a built-in experiment called "(In) elastic collision", it provided an ingenious way to calculate how high a ball bounces: Simply listen to them hit the floor and by the timing deduce heights. As pictured, you can see results from one trial and its series of decreasing bounce times and heights.

The third factor came easy: Height of drop. To make this obvious but manageable, I chose a broad range of 3 versus 6 feet.

The fourth and final factor occurred to me while washing dishes. We recently purchase a thick rubber mat for easy cleanup and comfortable standing in front of our sink. I realized that this would provide a good contrast to our hardwood floors for bounce height, the softer surface being obviously inferior.

To recap, the four factors and their levels I tested were:

A. Ball type: Hollow or Solid
B. Temperature: Freezer vs Room
C. Drop height: 6 vs 3 feet
D. Floor surface: Hardwood vs Rubber

Using Design-Expert® software (DX) I then laid out a two-level, full factorial of 16 runs in random order. To be sure of temperature being stabilized, I did only one run per day, recording the time the first bounce and its height (calculated by the Phypox boffins). For more experimental details, see the Appendix. You may enjoy reproducing the experiment on your own or as a science project.

When I completed the experiment and analyzed the results using DX, I was astounded to see that neither the type of ball nor the differing surfaces produced significant main effects. That made no sense based on my initial demonstrations on side-by-side bounce for the two balls on the floor versus the rubber mat.

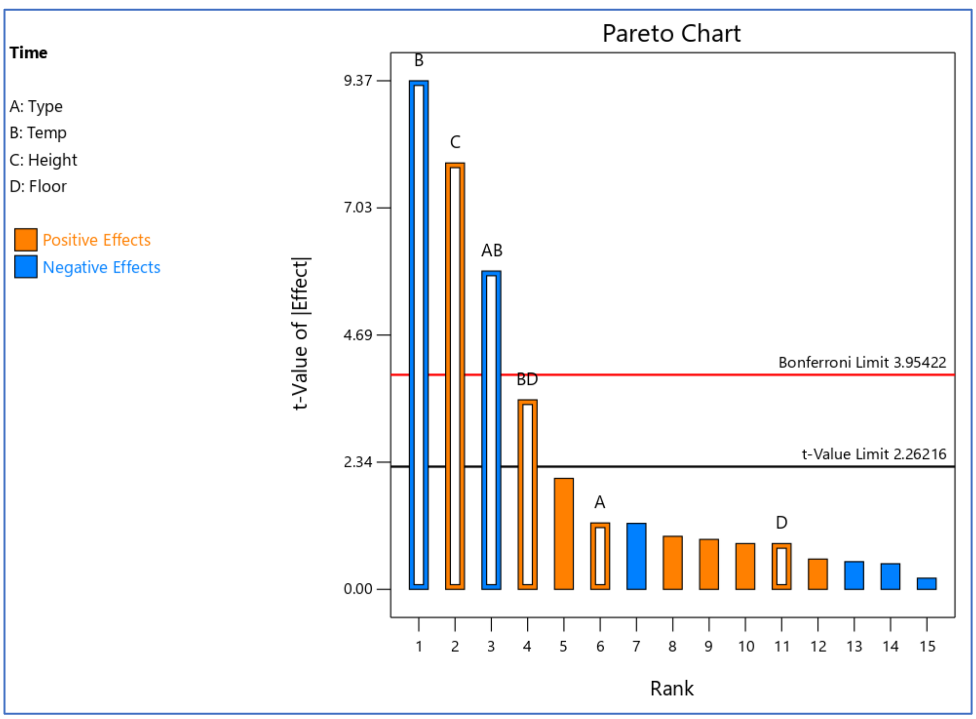
Keeping in mind that my experiment provided a multifactor test of two other variables, perhaps you can guess what happened. I will give you a hint: Factors often interact to produce surprising results, such as time and temperature suddenly coming together to create a fire (or as I would say as a chemical engineer—an "exothermic reaction").

Read on for the results on my elastic spheroid experiment to see how the factors interacted in delightful ways. When laid out with DX graphics these make perfect sense even for non-physicists.

Part Two—Results

Design-Expert provides the astonishing result from data provided in the Appendix: Neither the type of ball (factor A) nor the differing surfaces (factor D) produced significant main effects on first-bounce time (directly related to height per physics). I will now explain.

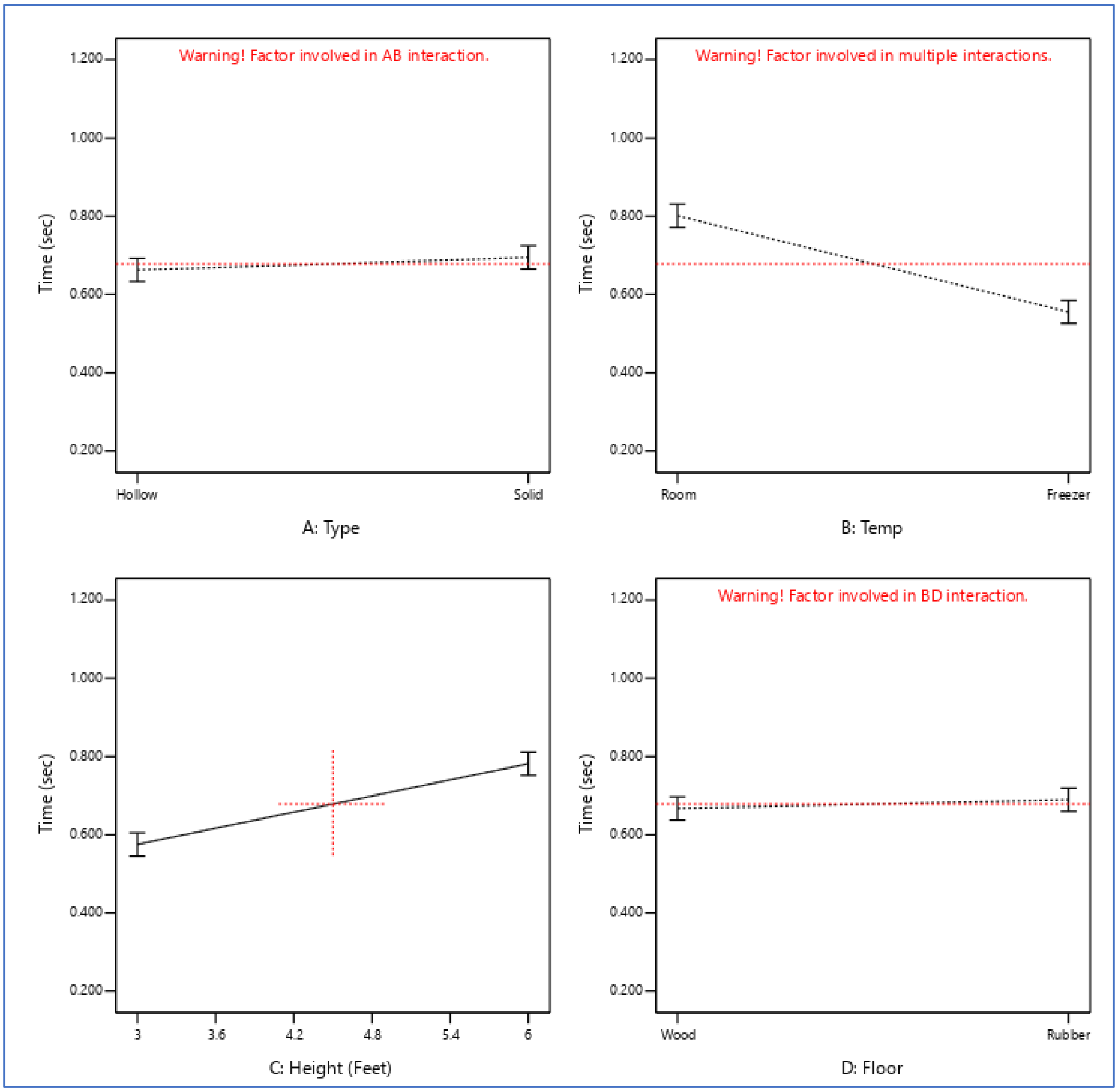
Let's begin with the Pareto Chart of effects on bounce time (scaled to t-values).



First observe the main effects of A (ball type) and D (floor surface) falling far below the t-Value Limit: They are insignificant (p>>0.05). Weird!

Next, skipping by the main effect of factor B (temperature) for now (I will get back to that shortly), notice that C—the drop height—towers high above the more conservative Bonferroni Limit: The main effect is very significant. The orange shading indicates that increasing drop height creates a positive effect—it increases the bounce time. This makes perfect sense based on physics (and common knowledge).

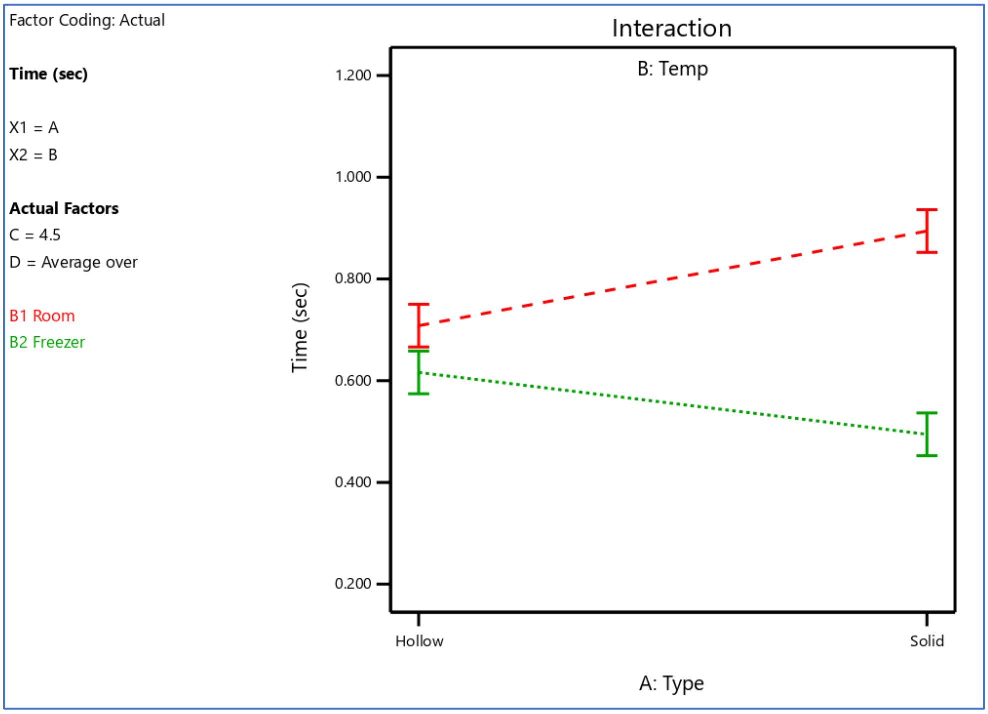
Now look at a multi-view Model Graphs for all four main effects.



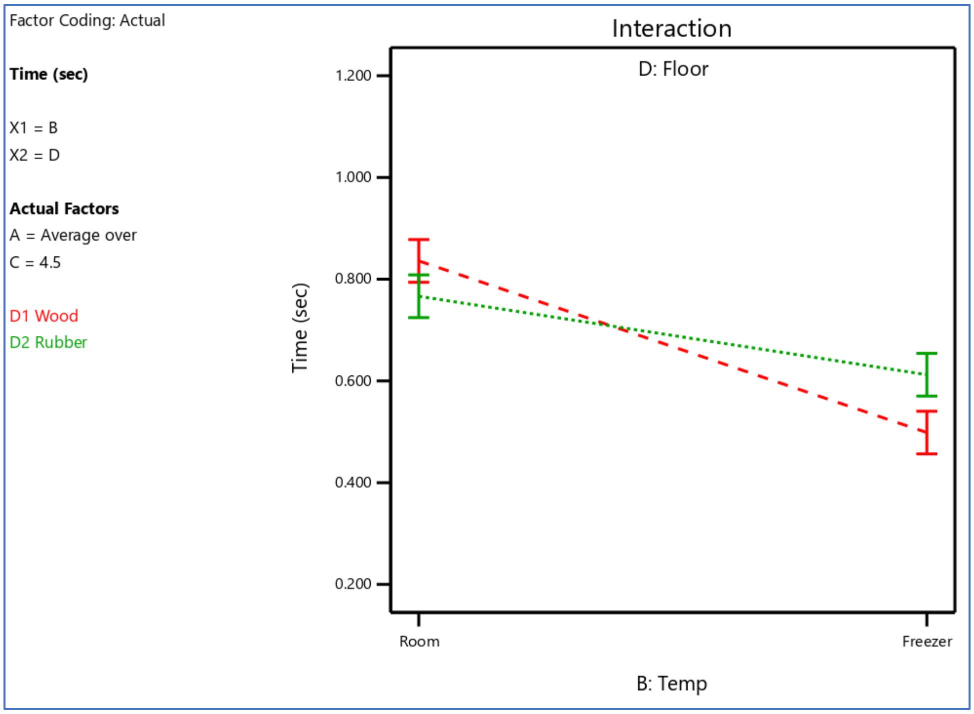
The plot at the lower left shows how the bounce time increased with height. The least-significant-difference 'dumbbells' at either end do not overlap. Therefore, the increase is significant ($p<0.05$). The slope quantifies the effect—very useful for engineering purposes.

However, as DX makes clear by its warnings, the other three main effects, A, B and D, must be approached with great caution because they interact with each other. The AB and BD interactions will tell the true story of the complex relationship of ball type (A), their temperature (B) and the floor material (D).

See by the interaction plot how the effect of ball type depends on the temperature. At room temperature (the top red line), going from the hollow to the solid ball produces a significant increase in bounce time. However, after being frozen, the balls behaved completely opposite—hollow beating solid (bottom green line). These opposing effects caused the main effect of ball type (factor A) to cancel!



Incredibly (I've never seen anything like this!), the same thing happened with the floor surface: The main effect of floor type got washed out by the opposite effects caused by changing temperature from room (ambient) to that in the freezer (below 0 degrees F).



Conclusion

Changing one factor at a time (OFAT) in this elastic spheroid experiment leads to a complete fail. Only by going to the *multifactor* testing approach of statistical DOE (design of experiments) can researchers reveal breakthrough interactions. Furthermore, by varying factors in parallel, DOE reveals effects far faster than OFAT.

If you still practice old-fashioned scientific methods, give DOE a try. You will surely come out far ahead of your OFAT competitors.

References

1. "Magic of multifactor testing revealed by fun physics experiment: Part One—the setup", Mark J. Anderson, StatsMadeEasy blog, 8/23/20, www.statsmadeeasy.net/2020/08/magic-of-multifactor-testing-revealed-by-fun-physics-experiment-part-one-the-setup/.
2. "Magic of multifactor testing revealed by fun physics experiment: Part Two—the amazing results", Mark J. Anderson, StatsMadeEasy blog, 8/31/20, www.statsmadeeasy.net/2020/08/magic-of-multifactor-testing-revealed-by-fun-physics-experiment-part-two-the-amazing-results/.
3. "Bouncy Ball Science Experiment: Does the Temperature of a Ball Affect its Bounce?", Frugal Fun for Boys and Girls, 3/13/17, https://frugalfun4boys.com/bouncy-ball-science-experiment-temperature-ball-affect-bounce/.
4. "Three Science Experiments You Can Do With Your Phone", Rhett Alain, *Wired*, 8/16/18, www.wired.com/story/phone-science-experiments/.
5. "Experiment: Inelastic Collision", Phyphox, https://phyphox.org/wiki/index.php/Experiment:_Inelastic_Collision.

Appendix: Experimental Details

Detail on factors:

A. Ball type (bought for $3.50 each from Five Below (www.fivebelow.com)):
    1. 4 inch, 41 g, hollow, licensed (Marvel Spiderman) playball from Hedstrom (Ashland, OH)
    2. 4 inch, 159 g, energy high bounce ball from PPNC (Yorba Linda, CA)

B. Temperature (equilibrated by storing overnight or longer):
    1. Freezer at about -4 F
    2. Room at 72 to 76 F with differing levels of humidity

C. Drop height (released by hand):
    1. 3 feet
    2. 6 feet

D. Floor surface:
    1. Oak hardwood
    2. Rubber, 3/4" thick, Anti Fatigue Comfort Floor Mat by Sky Mats (www.skymats.com)

Measurement:

Measurements done with Android PhyPhox app "(In)Elastic". Record T_1 and H_1, time and height (calculated) of first bounce. As a check note H_0, the estimated drop height—this is already known (specified by factor C low and high levels).

Data:

Std #	Run #	A: Ball type	B: Temp deg F	C: Height feet	D: Floor type	Time seconds	Height centimeters
1	16	Hollow	Room	3	Wood	0.618	46.85
2	6	Solid	Room	3	Wood	0.778	74.14
3	3	Hollow	Freezer	3	Wood	0.510	31.91
4	12	Solid	Freezer	3	Wood	0.326	13.02
5	8	Hollow	Room	6	Wood	0.829	84.33
6	14	Solid	Room	6	Wood	1.119	153.54
7	1	Hollow	Freezer	6	Wood	0.677	56.17
8	4	Solid	Freezer	6	Wood	0.481	28.34
9	5	Hollow	Room	3	Rubber	0.598	43.92
10	10	Solid	Room	3	Rubber	0.735	66.17
11	2	Hollow	Freezer	3	Rubber	0.559	38.27
12	7	Solid	Freezer	3	Rubber	0.478	28.03
13	15	Hollow	Room	6	Rubber	0.788	76.12
14	11	Solid	Room	6	Rubber	0.945	109.59
15	9	Hollow	Freezer	6	Rubber	0.719	63.43
16	13	Solid	Freezer	6	Rubber	0.693	58.96

Observations:

- Run 7: First drop produced result >2 sec with height of 494 cm. This is >16 feet! Obviously something went wrong. My guess is that the mic on my phone is having trouble picking up the sound of the softer solid ball and missed a bounce or two. In any case, I redid the bounce.
    - Starting run 8, I will record Height 0 in Comments as a check against bad readings.
- Run 8: Had to drop 3 times to get time registered due to such small, quiet and quick bounces.
    - Could have tried changing setting for threshold provided by the (In)Elastic app.
- Run 14: Showing as outlier for height so it was re-run. Results came out nearly the same 1.123 s (vs 1.119 s) and 154.62 cm (vs 153.54). After transforming by square root these results fell into line. This makes sense by physics being that distance for is a function of time squared.

Suggestions for future:

- Rather than drop the balls by eye from a mark on the wall, do so from a more precise mechanism to be more consistent and precise for height
- Adjust up for 3/4" loss in height of drop due to thickness of mat
- Drop multiple times for each run and trim off outliers before averaging (or use median result)
- Record room temp to nearest degree