ROBUST DESIGN -- REDUCING TRANSMITTED VARIATION:

FINDING THE PLATEAUS VIA RESPONSE SURFACE METHODS

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ABSTRACT

This paper is an overview of the propagation of error technique for robust design. It demonstrates the practical application of response surface methods, augmented by propagation of error, to solve a tough manufacturing problem and improve product quality.

PAPER

Robust design aims at making a process less sensitive to variation in the input factors. To accomplish this you should set controllable factors to levels that reduce variation in the response:
1) Caused by variation in the uncontrollable factors (Taguchi 1979); and 2) Transmitted from variation in the controllable factors. In this paper we focus on reducing transmitted variation using the propagation of error (POE) technique.

POE is a tool to find controllable factors settings that maximize quality, which we define as making a product to target with minimum variation. It requires construction of mathematical models via response surface methods (RSM) (Box and Draper 1987). Figure 1 illustrates a typical process we want to improve. Using the RSM and the POE techniques, we seek levels of the controllable factors that center response values on their respective targets while simultaneously reducing variation transmitted to the response from variation (lack-of-control) in the controllable factors.

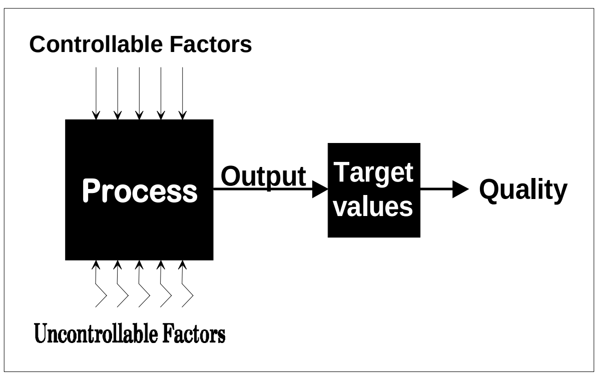

Figure 1: Process Schematic

When RSM reveals curvilinear relationships between controllable factors and responses, transmitted variation can be reduced by moving to plateaus. For example, in the case shown by Figure 2a, moving the control factor setting from level A to B will result in a more robust design.

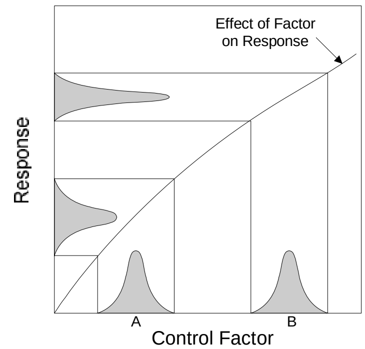

Figure 2a: Transmitted Variation Dependent on Factor Level

Linear relationships between controllable factors and responses, as seen in Figure 2b, give us factors that can be used to adjust nominal values of the response without affecting the transmitted variance.

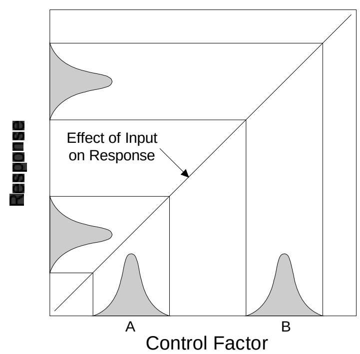

Figure 2b: Transmitted Variation Independent of Factor Level

To illustrate the use of POE, let's analyze a power circuit design (Taguchi 1979). The goal is to design a circuit with an output voltage of 115 volts from a fixed input voltage. Figure 3 is a simple schematic of the elements of the power circuit under consideration.

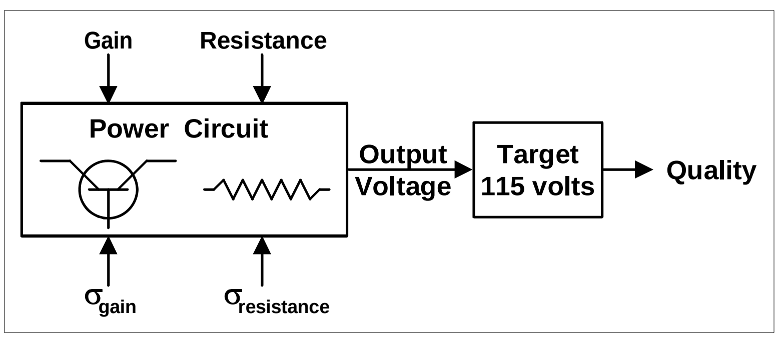

Figure 3: Power Circuit

Consider two controllable factors:

1) Transistor Gain -- output voltage is proportional to gain.
2) Resistance -- output voltage is inversely proportional to resistance.

The variation in gain and resistance about their nominal values cannot be controlled. Assume that both variances stay constant over the range of nominal values being considered. Figure 4 shows the relationship between output voltage and transistor gain while holding resistance at 100 ohms.

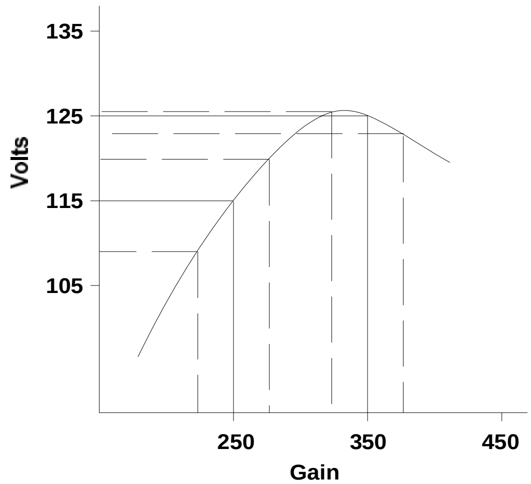

Figure 4: Relationship Between Gain and Output Voltage

A transistor gain of 250 produces the desired output voltage of 115 volts. By changing to a transistor with a gain of 350 the designer can reduce the transmitted variation. Note that this design change increases the nominal output voltage well above the desired level of 115 volts. However, the output voltage can be adjusted to target by simply increasing the resistance from 100 ohms to 500 ohms as shown in Figure 5.

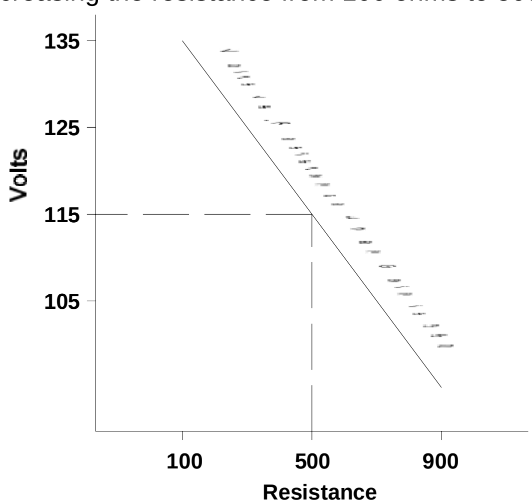

Figure 5: Using Resistance to Control Output Voltage

Because the relationship between output voltage and resistance is linear, changing the resistance does not change the transmitted variance. In the case of the power circuit, robust design principles produce an on-target response (a nominal output voltage of 115 volts) with less variation.

Let's look at propagation of error as a mathematical tool for reducing transmitted variance. First we need to quantify the relationship between a response and the controllable factors. Response surface methods will do this by approximating the relationship with a polynomial. The variation transmitted to the response can then be modeled by taking the partial derivatives of the polynomial with respect to the controllable factors.

$$\sigma_Y^2 = \sum_{i=1}^{k} \left(\frac{\partial Y}{\partial X_i} \right)^2 \sigma_{X_i}^2 + \sigma_{resid}^2 \qquad (1)$$

For example, assume that a response that depends on one controllable factor can be adequately modeled with a simple quadratic polynomial (equation 2). The actual coefficients are indicated in equation 3. Taking the partial derivative with respect to the independent factor (equation 4) provides the model for propagation of error (equation 5).

$$\hat{Y} = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2 \quad (2)$$

$$\hat{Y} = 15 + 25x_1 - 0.7x_1^2 \quad (3)$$

$$\sigma_Y^2 = \left(\frac{\partial Y}{\partial x}\right)^2 \sigma_x^2 + \sigma_{resid}^2 \quad (4)$$

$$\sigma_Y = \sqrt{(25 - 1.4x_1)^2 \sigma_x^2 + \sigma_{resid}^2} \quad (5)$$

The transmitted variation can now be calculated by substituting the variance in the independent factor, the residual variance (noise) and taking the square root, as shown in equation 5. Figure 6 shows response Y (left axis) and the transmitted variance σ_Y (right axis), assuming a σ_x of 1 and a σ_{resid} of 0.

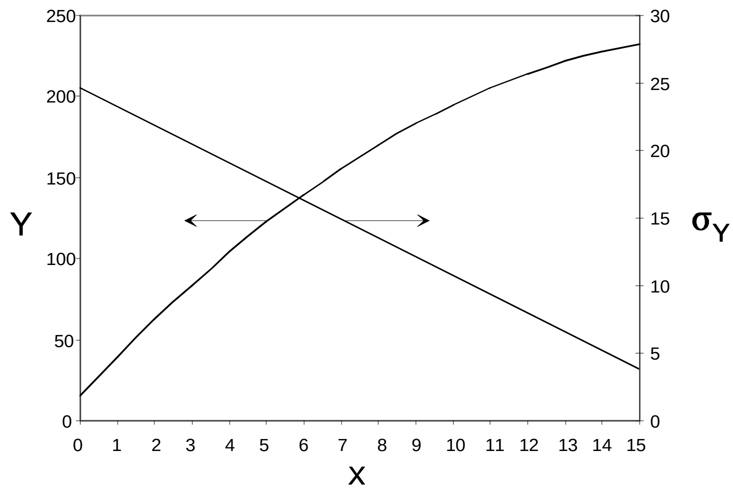

Figure 6: Error Transmitted by a Quadratic Response Function

In general, the propagation of error technique shows how to reduce variation transmitted to the response from variation in the controllable factors. It depends on:

1. Boundaries of the factor space explored.
   - The model must adequately represent actual behavior.
   - There must be significant curvature within the boundaries.
2. The order of the polynomial model for response.
   - Non-linear factors provide opportunities to find plateaus.
   - Linear factors allow us to adjust nominal values to target.
3. Nature of variation in controllable factors.
   - As illustrated in Figure 7a, if the variation is independent of the size of the controllable factor level, it can be adjusted to reduce the transmitted variation.
   - If the variation is a percentage of the size of the controllable factor level (rather than a constant), changing the value of the controllable factor may not change the transmitted variation. An example is shown in Figure 7b.

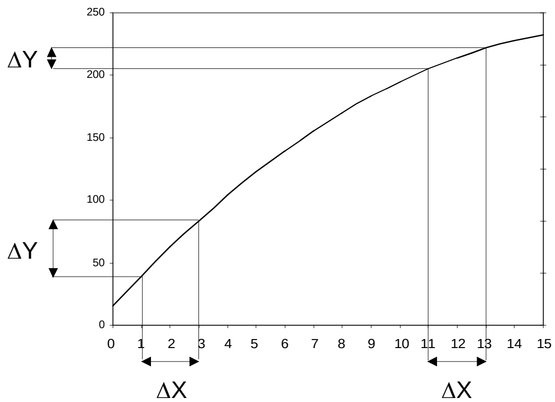

Figure 7a: Constant Error

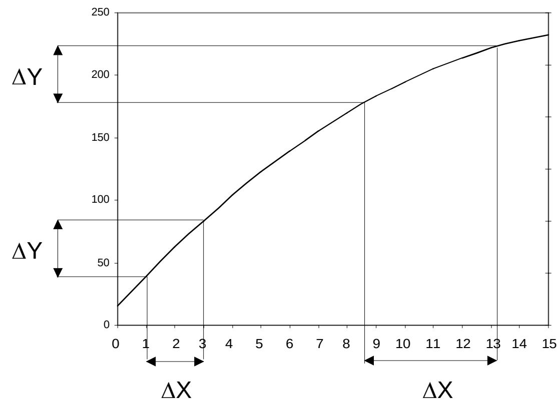

Figure 7b: Percentage Error

To illustrate the use of propagation of error, let's look at the problem of holding nominal values on an automated lathe. We will study the process and try to reduce deviations from nominal dimensions. The investigation will study three key factors using response surface methods.

Factor	Range	Units
Speed (cutting speed)	330. - 700.	fpm
Feed (feed rate)	0.010 - 0.022	ipr
Depth (depth of cut)	0.05 - 0.10	inches

Table 1: Region of Interest for Lathe Study

Table 1 lists the region of interest for each of the three factors. We want a design to fit a quadratic model. In this situation a Box-Behnken (1960) design will be a reasonable choice for the experiments. (Box-Behnken designs are space-filling designs, requiring only three levels per factor.)

Table 2 shows the results from the experiment. The response, labeled "delta," gives the deviation of the finished part's dimension from its nominal value in mils (0.001 inches).

Std Ord	Run Ord	A Speed fpm	B Feed ipr	C Depth inches	Y1 delta mils
1	17	330	0.010	0.075	-0.081
2	1	700	0.010	0.075	-0.410
3	6	330	0.022	0.075	-0.240
4	3	700	0.022	0.075	0.530
5	14	330	0.016	0.050	-0.280
6	4	700	0.016	0.050	0.120
7	16	330	0.016	0.100	0.520
8	11	700	0.016	0.100	0.310
9	2	515	0.010	0.050	-0.240
10	10	515	0.022	0.050	0.097
11	13	515	0.010	0.100	0.140
12	15	515	0.022	0.100	0.370
13	12	515	0.016	0.075	-0.130
14	7	515	0.016	0.075	-0.071
15	9	515	0.016	0.075	-0.060
16	5	515	0.016	0.075	-0.190
17	8	515	0.016	0.075	-0.140

Table 2: Data from Lathe Experiment (Box-Behnken RSM design)

Regression analysis reveals a significant quadratic model (shown in terms of coded factors, where -1 indicates the low factor level and +1 the high):

$$delta = -0.118 + 0.079 *A + 0.168 *B + 0.205 *C + 0.072 *A^2 - 0.004 *B^2 + 0.214 *C^2 + 0.275 *AB - 0.153 *AC - 0.027 *BC \quad (6)$$

As seen in Figures 8a, 8b and 8c many combinations of the three controllable factors will give an average delta of zero. (Look for the contour labeled “0.0”, zero.)

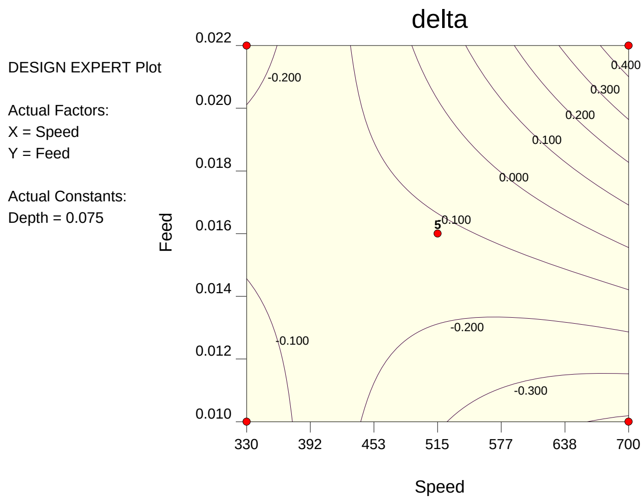

Figure 8a: Contour Plot for Delta with Factor C set at Midpoint

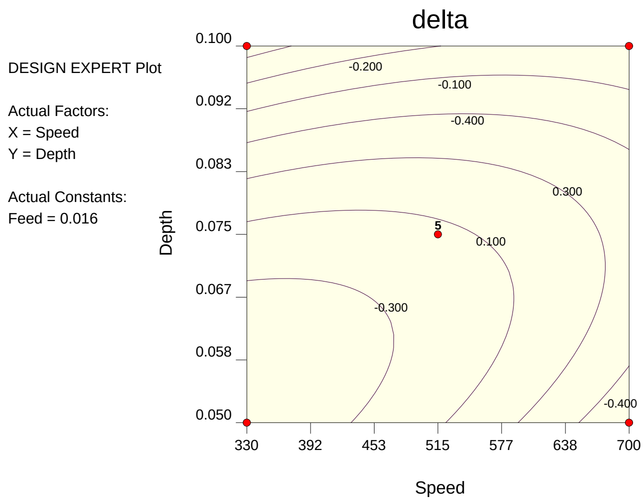

Figure 8b: Contour Plot for Delta with Factor B set at Midpoint

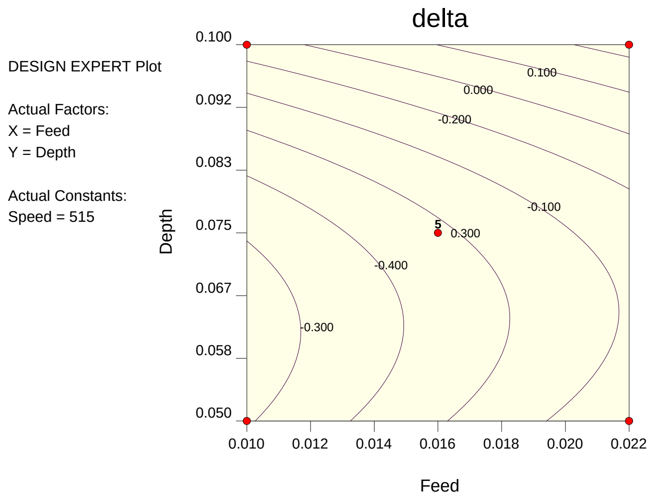

Figure 8c: Contour Plot for Delta with Factor A set at Midpoint

Robust design can now be applied to decide which combination will be most reliable. Not only must the average dimension be correct, but each part must be as close to the nominal as possible. The latter objective can be accomplished by reducing the variation transmitted by lack of control of the controllable factors. We will use propagation of error to find the robust operating conditions. Table 3 shows the expected variation of the controllable factors about their set points.

Factor	Standard deviation
A - Speed	5 fpm
B - Feed	0.003 ipr
C - Depth	0.0125 inches
Residual standard deviation = 0.075 mils	

Table 3: Expected Variation of Controllable Factors and Residual Error

Propagation of error analysis produces Figures 9a, 9b and 9c. (These correspond to Figures 8a, 8b and 8c above). Look for conditions that minimize transmitted error.

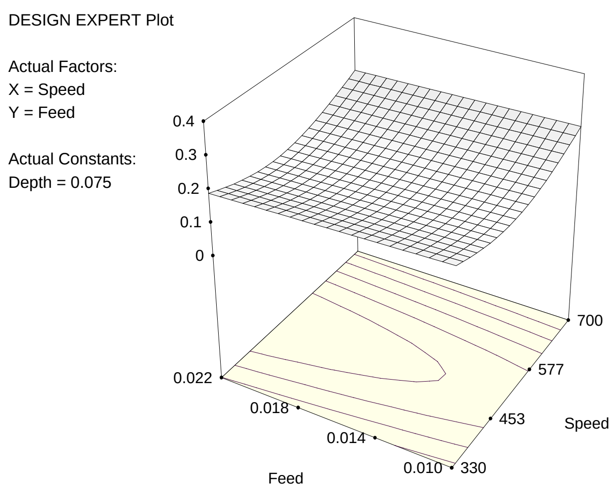

Figure 9a: Surface Plot for Sigma with Factor C set at Midpoint

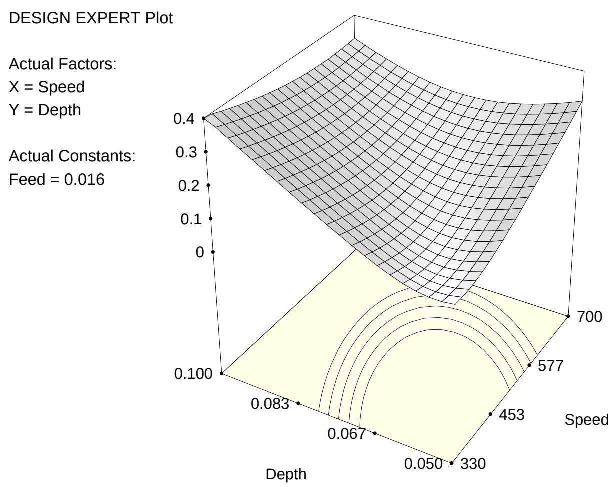

Figure 9b: Surface Plot for Sigma with Factor B set at Midpoint

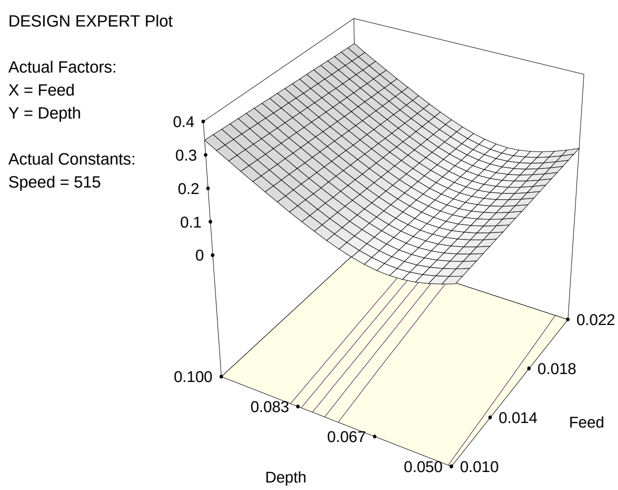

Figure 9c: Surface Plot for Sigma with Factor A set at Midpoint

With the aid of computer software (Helseth, et al., 1994) that makes use of a numerical search technique (Derringer and Suich 1980), we search for controllable factor settings that simultaneously achieve a delta of zero, while minimizing the transmitted variation, σ(delta). Table 4 lists the most desirable operating conditions and the predicted results:

Variable	Final Setting
A - Speed	511 fpm
B - Feed	0.022 ipr
C - Depth	0.065 inches
delta = 0.000 mils	
σ(delta) = 0.112 mils	

Table 4: Best Operating Conditions

These conditions represent a compromise that brings the nominal dimension as close to zero as possible, while minimizing the transmitted variation. The optimum conditions (speed = 511 fpm and depth

= 0.065 inches) are shown by flags on the contour plots (sliced through feed = 0.022 ipr) of delta and σ(delta) in Figures 10a and 10b.

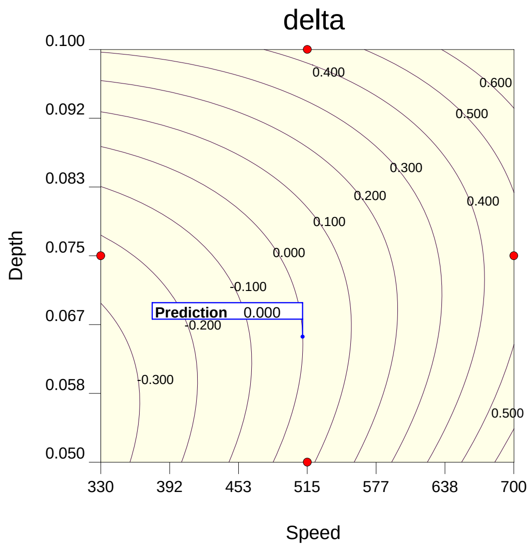

Figure 10a: Delta (in mils)

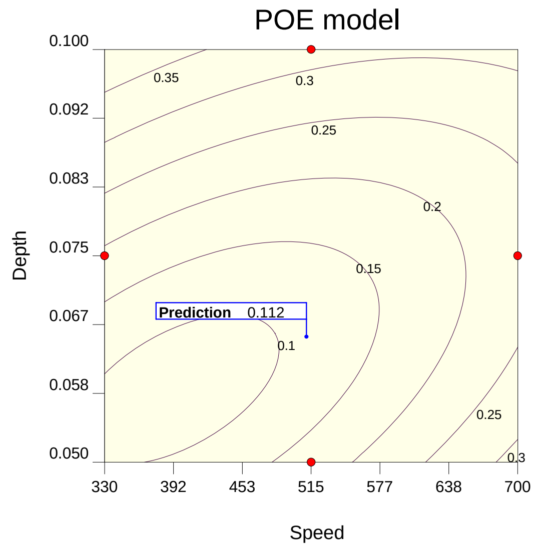

Figure 10b: Sigma of Delta (in mils)

For this part on the automated lathe, response surface methods augmented by propagation of error make a powerful combination for maximizing product quality. It results in a product made to target with the least amount of variation.

CONCLUSION

Using propagation of error adds a new dimension -- robust design -- to response surface methods. Not only do we learn to make the right product (achieve the targets for the responses), we also simultaneously minimize variation in the product. By making the process more robust to variation in the controllable factors, we improve product quality and reliability.

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